Exercise 32

For the following exercises, solve the equations over the complex numbers.

$$x^2 + 8x + 25 = 0$$

Solution

The two terms with x, x^2 and $8x$, cannot be combined, so it's necessary to complete the square to solve for x. Recall the following algebraic identity.

$$(x + B)^2 = x^2 + 2xB + B^2$$

Notice that $2B = 8$, which means $B = 4$ and $B^2 = 16$. Add and subtract 16 from the left side and apply the identity.

$$(x^2 + 8x + 16) + 25 - 16 = 0$$

$$(x + 4)^2 + 9 = 0$$

Now that x appears in only one place, it can be solved for. Subtract 9 from both sides.

$$(x + 4)^2 = -9$$

Take the square root of both sides.

$$\begin{aligned}\sqrt{(x + 4)^2} &= \sqrt{-9} \\ &= \sqrt{9(-1)} \\ &= \sqrt{9}\sqrt{-1} \\ &= 3i\end{aligned}$$

Since there's an even power under an even root, and the result is to an odd power, an absolute value sign is needed around $x + 4$.

$$|x + 4| = 3i$$

Remove the absolute value sign by placing $\pm$ on the right side.

$$x + 4 = \pm 3i$$

Subtract 4 from both sides.

$$x = -4 \pm 3i$$

Therefore, $x = \{-4 - 3i, -4 + 3i\}$.